Amnesiac Flooding: Synchronous Stateless Information Dissemination

Volker Turau

47th Int. Conference on Current Trends in Theory and Practice of Computer Science

January 25th, 2021

Institute of Telematics
Hamburg University of Technology
TUHH

1

Introduction

Stateless Protocols

Definition (Stateless Protocol)

A *stateless protocol* is a communications protocol in which no session information is retained by participating nodes.

- Stateless protocols do not utilize local storage
- Big advantage in high volume applications, increasing performance by removing the load caused by retention of session information

Information Dissemination Task

Given: A graph $G = (V, E)$, $v_0 \in V$, and a message m
Task: Disseminate m to all nodes of V

Deterministic Flooding

- Deterministic Flooding
  - Originator v_0 sends message to all neighbors
  - A node receiving message for first time sends it to all its neighbors
- Properties
  - stateful algorithm
    - each node keeps a record of which messages have already arrived
  - storage per node $\approx$ number of disseminated messages
  - nodes cannot detect termination $\Rightarrow$ required storage grows over time
  - termination in $\epsilon_G(v_0) + 1$ rounds ($\epsilon_G(v_0)$ is maximal distance of v_0 to any other node)

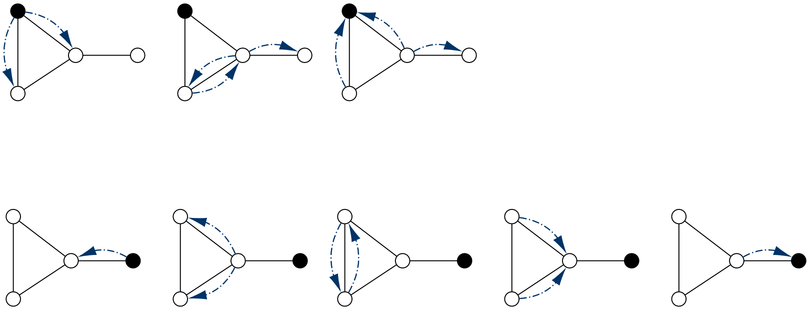
Amnesiac Flooding

- Stateless variant of flooding by Hussak & Trehan [PODC19]
- Only for synchronous systems

Every time a node receives a message, it forwards it to those neighbors from which it didn’t receive message in current round

- Difference to classic flooding, a node may forward a message several times

Amnesiac Flooding: Examples



Simple observation: Amnesiac flooding on bipartite graphs terminates in $\epsilon_G(v_0)$ rounds

Amnesiac Flooding: Results

- $Flood_G(v_0)$ denotes number of rounds of $\mathcal{A}_{AF}$ until termination
- $\mathcal{A}_{AF}$ terminates on any finite graph
  - If G is bipartite graphs: $Flood_G(v_0) = \epsilon_G(v_0)$
  - Non-bipartite graphs: $Flood_G(v_0) \leq \epsilon_G(v_0) + Diam(G) + 1$
  - Bound is sharp

Contributions

- Reduction of amnesiac flooding on general graphs to bipartite graphs
    - Intuitive insights into amnesiac flooding
    - Simplifies proofs
- Extension of single-source flooding to multi-source flooding
- Sharp upper and lower bounds for termination time for multi-source flooding
- Which k-node subset is an optimal source for amnesiac flooding

2

Singe-Source Flooding

The Auxiliary Graph $\mathcal{G}(v_0)$

Definition

Denote by $\mathcal{F}(v_0)$ the subgraph of G with node set V and all edges of G that are not cross edges with respect to v_0.

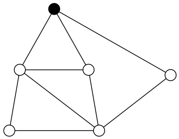
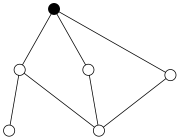


Figure: Graph G

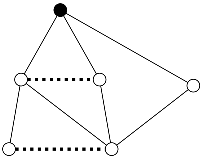


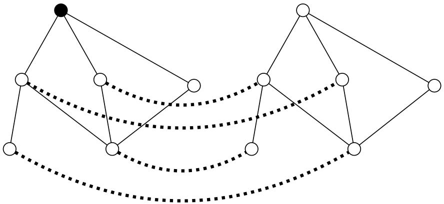
Subgraph $\mathcal{F}(v_0)$

The Auxiliary Graph $\mathcal{G}(v_0)$

Definition

Let $\mathcal{G}(v_0)$ be the graph consisting of two copies of $\mathcal{F}(v_0)$ with node sets V and V' and additionally for any cross edge (u, w) of G the edges (u, w') and (w, u').





Lemma

$\mathcal{G}(v_0)$ is bipartite.

The Auxiliary Graph $\mathcal{G}(v_0)$

Lemma

A node v receives a message from w in round i in G iff in round i node v receives a message from w in $\mathcal{G}(v_0)$, or v' receives a message from w or from w' in $\mathcal{G}(v_0)$.

Theorem

$Flood_G(v_0) = Flood_{\mathcal{G}(v_0)}(v_0)$ *for every* $v_0 \in V$.

Corollary

$Flood_G(v_0) = \epsilon_{\mathcal{G}(v_0)}(v_0) \leq \epsilon_G(v_0) + Diam(G) + 1.$

3

Multi-Source Flooding

Amnesiac Flooding

Algorithm 1: Algorithm $\mathcal{A}_{AF}$ distributes message m in the graph G

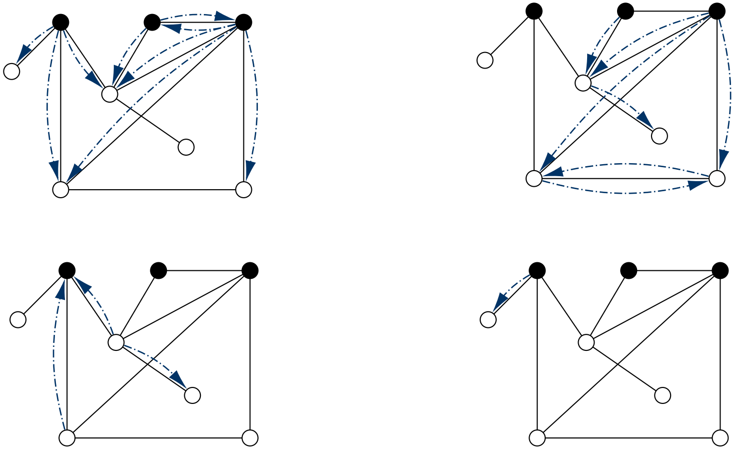
input : A graph $G = (V, E)$, a subset S of V, and a message m

Initially: Each node $v \in S$ sends m to each neighbor;

Each node v executes

```
M := N(v);
foreach receive(w, m) do
    M := M \ {w}
if M ≠ N(v) then
    forall u ∈ M do send(u, m);
```

Amnesiac Flooding: Example



Multi-Source Flooding

Definition

For $S \subseteq V$ denote by $Flood_G(S)$ the number of rounds algorithm $\mathcal{A}_{AF}$ requires to terminate when started by all nodes in S.

Theorem

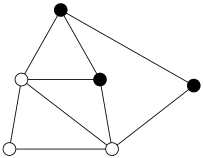
Let $G = (V, E)$ be a connected graph. For every $S \subseteq V$ there is a bipartite graph $\mathcal{G}(S)$ with a node v^* such that $Flood_G(S) = Flood_{\mathcal{G}(S)}(v^*) - 1$.

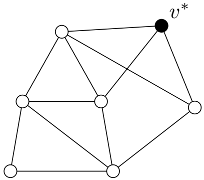
Corollary

$Flood_G(S) \leq d(S, V) + 1 + Diam(G)$

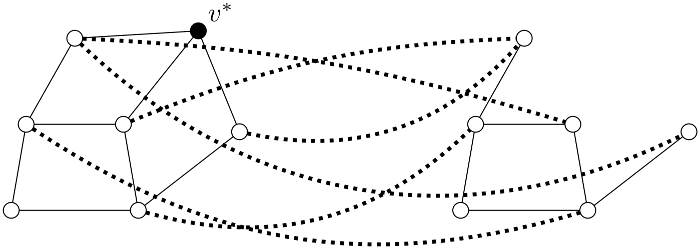
The Graph $\mathcal{G}(S)$

- Let $S \subseteq V$
- Virtual source v^*





The Graph $\mathcal{G}(S)$



4

Optimal Choice of Flooding Set

Optimal Choice of Flooding Set S

Definition

For $1 \leq k \leq n$ define $Flood_k(G) = \min\{Flood_G(S) \mid S \subseteq V \text{ with } |S| = k\}$

Theorem

Let $G = (V, E)$ be a connected graph.

1. *If $k > 1$ then $r_k(G) \leq Flood_k(G) \leq r_k^{ni}(G) + 1 \leq r_{\lfloor k/2 \rfloor}(G) + 1$.*
2. *$Flood_k(G) = 1$ iff $n = k$ or G is bipartite with $|V_1| = k$ or $|V_2| = k$.*
3. *$Flood_k(G) \leq 3$ if $k \geq n/2$.*

Bipartite Graphs

Theorem

Let $G = (V_1 \cup V_2, E)$ be a connected, bipartite graph. If $k \geq 1$ then

1. $Flood_k(G) = r_k(G)$ *iff G has a k-center S with either* $S \subseteq V_1$ *or* $S \subseteq V_2$
2. $Flood_k(G) \leq r_k(G) + 2$
3. *If* $k \leq \max(|V_1|, |V_2|)$ *then* $Flood_k(G) \leq r_k(G) + 1$

5

Conclusion

Conclusion & Outlook

- Construction of a bipartite graph $\mathcal{G}(S)$ such that executions of amnesiac flooding on G and $\mathcal{G}(S)$ are equivalent
- Upper and lower bounds for round complexity of multi-source amnesiac flooding
- Open problems related to amnesiac flooding
  - Conjecture: If G is connected, non-bipartite then $kFlood_k(G) \geq Rad(G) + k - 1$
  - $Flood_k(G)$ assumes one of three values in case G is bipartite. Is it possible to infer from structural parameters of G the value of $Flood_k(G)$?
  - Existence of a stateless asynchronous information dissemination algorithm

Amnesiac Flooding: Synchronous Stateless Information Dissemination

47th Int. Conference on Curre

Volker Turau

Professor

Phone +49 / (0)40 428 78 3530

e-Mail turau@tuhh.de

http://www.ti5.tu-harburg.de/staff/turau