Synchronous Concurrent Broadcasts for Intermittent Channels with Bounded Capacities

Volker Turau

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Institute of Telematics
Hamburg University of Technology
TUHH

Information Dissemination

Single-Source Broadcast

Given: A graph $G = (V, E)$, $v_0 \in V$, and a message m located at v_0

Task: Disseminate m to all nodes of V

Multi-Source Broadcast

Given: A graph $G = (V, E)$, $S \subset V$, and a message m located at the nodes of S

Task: Disseminate m to all nodes of V

Multi-Message Broadcast

Given: A graph $G = (V, E)$, message $m_1, \ldots m_s$ located at the nodes $v_1, \ldots, v_s$

Task: Disseminate $m_1, \ldots, m_s$ to all nodes of V

Single-Source Broadcast: Flooding

- Deterministic Flooding
  - Originator sends message *m* to all neighbors
  - Nodes receiving *m* for the first time, send it to all neighbors
  - Flooding is a stateful algorithm
    - Each node keeps a record of which messages have already been received
- Probabilistic Flooding
  - Stateless algorithm
  - Messages are forwarded to neighbors based on a probability
  - Expected number of messages is reduced
- Amnesiac Flooding [PODC19]
  - Stateless deterministic algorithm (synchronous systems)
  - Every time a node receives *m*, it forwards *m* to those neighbors from which it didn't receive *m* in current round

Amnesiac Flooding

Algorithm 1: Algorithm $\mathcal{A}_{AF}$ distributes a message in the graph G

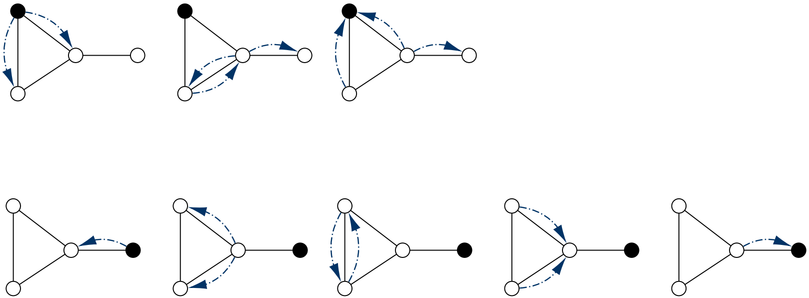
input : A graph $G = (V, E)$, $v_0 \in V$, and a message m

Round 1: v_0 sends m to each neighbor;

Round $i > 1$: Each node v executes

```
    M := N(v);
    foreach receive(w, m) do
        M := M \ {w}
    if M ≠ N(v) then
        forall u ∈ M do send(u, m);
```

Amnesiac Flooding: Examples



- $\mathcal{A}_{AF}$ terminates and each message is sent at most twice per edge

Questions

- Can $\mathcal{A}_{AF}$ be used for multi-source and multi-message broadcast?
- Yes, if channel capacities are unbounded
- For bounded channel capacities messages must be backed up and send later
- Does $\mathcal{A}_{AF}$ terminate in this case?
- Idea: Bounded channels are modelled by intermitted channels

Amnesiac Flooding

- Crucial for termination of $\mathcal{A}_{AF}$:
  - Forwarding of messages is always performed in round immediately following reception
  - $\mathcal{A}_{AF}$ no longer terminates when message forwarding is suspended for some rounds

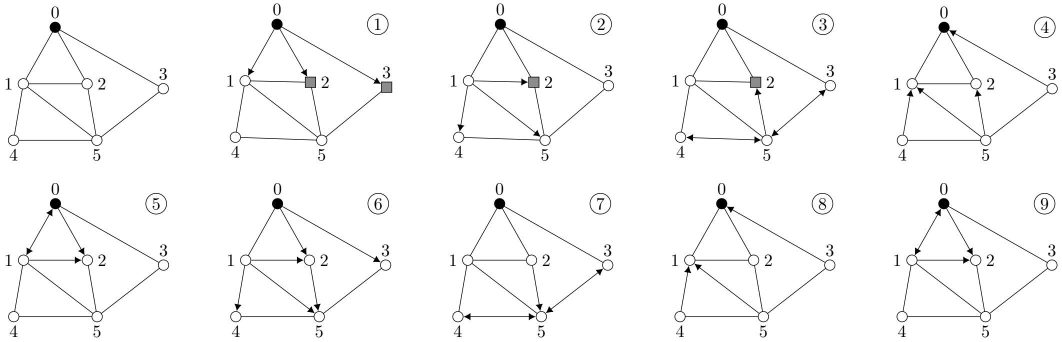


Figure: Node 2 cannot send messages in rounds 1, 2, 3 and node 3 not in round 1

Contributions

- Algorithm $\mathcal{A}_{AFI}$:
  - Extension of $\mathcal{A}_{AF}$ to cope with a limited number of channel suspensions
- Proof that $\mathcal{A}_{AFI}$ is correct for multi-source broadcasting
- New algorithm for multi-message broadcast

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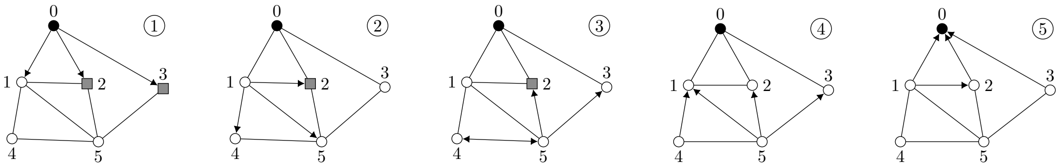
Intermittent Channels

Intermittent Channels

- Basic idea of $\mathcal{A}_{AFI}$
  - If *m* can't be forwarded in current round, it is postponed until next available round with **same parity**
  - If blocked round is odd (resp. even), *m* will be forwarded in next available odd (resp. even) round



Round numbers indicate round of reception

Algorithm $\mathcal{A}_{AFI}$

Algorithm 2: Algorithm $\mathcal{A}_{AFI}$ distributes a message m in the graph G

Initialization

$parity := \text{true};$

$M[true] := M[false] := \perp;$

Each node v executes in every round

Upon receiving message m from w:

$M[parity].add(w);$

if channel is available and $M[parity] \neq \perp$ **then**

forall $u \in N(v) \setminus M[parity]$ **do** send(u, m);

$M[parity] := \perp;$

$parity := \neg parity;$

function broadcast(m)

$M[parity] := \emptyset;$

Algorithm $\mathcal{A}_{AFI}$

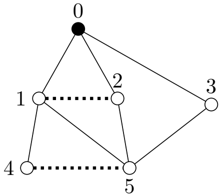
Theorem

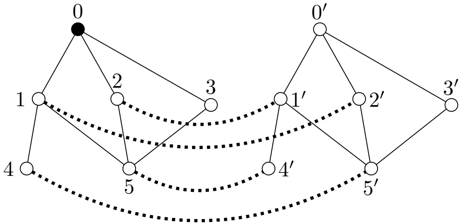
Let G be a graph, A an availability scheme for G, and $f = |\{(v,i) \mid A(v,i) = false\}|$.

$\mathcal{A}_{AFI}$ *delivers a message (resp. terminates) after at most* $Diam(G) + 2f$ *(resp.* $2Diam(G) + 2f + 1$*) rounds. If G is bipartite each message is forwarded* $|E|$ *times, otherwise* $2|E|$ *times.*

- Idea of proof:
  - For availability scheme A construct a directed bipartite graph $\mathcal{B}_A(v_0)$ such that execution of $\mathcal{A}_{AFI}$ on G with respect to A is equivalent to execution of amnesiac flooding $\mathcal{A}_{AF}$ on $\mathcal{B}_A(v_0)$
  - Starting point for construction of $\mathcal{B}_A(v_0)$ is the double cover $\mathcal{G}(v_0)$ of G

Double Cover $\mathcal{G}(v_0)$





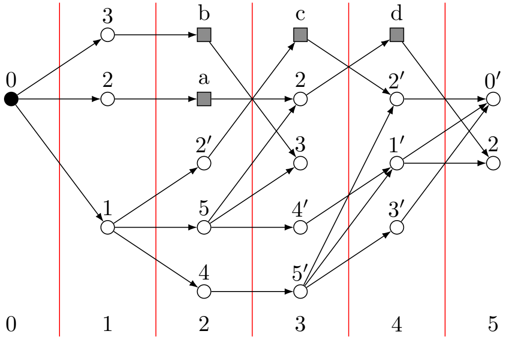
- Left: G: Dashed edges are cross edges (v_0 is broadcasting node)
- Right: $\mathcal{G}(v_0)$, dashed edges are the replacement edges
- Orientation: Top down, left to right

Predecessors of v in $\mathcal{G}(v_0)$ are copies of nodes in G that send in round i of $\mathcal{A}_{AF}$ a message to v and successors of v in $\mathcal{G}(v_0)$ receive a message from v in round $i+1$

The Graph $\mathcal{B}_A(v_0)$

- $\mathcal{G}(v_0)$ is *streched* over time
- $\mathcal{B}_A$ is defined layer by layer
- Nodes of $\mathcal{B}_A$ are of two different types
  - Copies of nodes of $\mathcal{G}(v_0)$ and
  - *dummy nodes*, they correspond to times when a channel is unavailable
- Execution of $\mathcal{A}_{AF}$ on $\mathcal{B}_A$
  - All nodes including dummy nodes behave according to original $\mathcal{A}_{AF}$
  - No intermitted channels

The Graph $\mathcal{B}_A(v_0)$



- Availability scheme A:
  $A(v_2, 1) = A(v_2, 2) = A(v_2, 3) = A(v_3, 1) =$ *false* and *true* otherwise.
- $\mathcal{B}_A$ for availability scheme A has four dummy nodes

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Conclusion

Conclusion

- Broadcast algorithm $\mathcal{A}_{AFI}$ for systems with intermittent channels
- While $\mathcal{A}_{AFI}$ is of interest on its own, it is the basis to solve the general task of multi-message broadcasts in systems with bounded channel capacities
- Full paper available at https://arxiv.org/abs/2011

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28th Int. Colloquium on St

Volker Turau

Professor

Phone +49 / (0)40 428 78 3530

e-Mail turau@tuhh.de

http://www.ti5.tu-harburg.de/staff/turau